OVERLAND FLOW MODELLING FOR INUNDATION PREDICTION

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SUMMARY

The paper illustrates how a 2D flow model can help in predicting inundation due to flood waves propagation from levee failures or bank overtopping. A complex framework is required in order to achieve a global management strategy in hydraulic safety design at regional and local level, and mathematical models aimed at real time forecasting and civil protection planning necessarily become part of it. In this framework, models should work so to predict the time of propagation of a 2D flood wave in overland flow, the water heights and velocities reached at each location and, crossing the results together with land use maps and economic activities data, to assess the impacts of inundation and to allow a comparative analysis of the interventions required for land protection. The basis for such a mathematical model lays in the 2D StVenant equations, that need to be integrated numerically. Classical numerical finite element methods, however, fail in describing 2D free surface fields of motion with large displacements and high velocities if compared with those in groundwater hydraulics. This seems to be due to the not respected mass balance in that fluxes are calculated along sides that mark a discontinuity in hydraulic head gradients. To overcome this problem some interesting techniques have been reported, that consider sides of a mesh staggered, with respect to the one used for calculating heads, thus allowing to calculate fluxes along sides that don’t have discontinuities in head gradients across. This allows a good representation of the motion field, and does not require a too complex numerical programming in comparison with classical Galerkin methods. On the other hand, all hydraulic singularities such as culverts, embankments, roads, and weirs, need to be taken in account so to represent a good approximation of real phenomena heavily affecting overland flow. Another key feature required to such a model is the possibility of coupling 1D and 2D propagation, that is of prime importance in overtopping simulation.

1.INTRODUCTION

While it is quite clear how to model one-dimensional propagation of flood waves, it is not such how to represent 2D motion, since there’s a wide group of classical numeric algorithms for the solution of StVenant equations, the well known finite elements methods, that fail in dealing with the governing mathematical system under the point of view of mass balance conservation. This is due to the algorithm that supposes a discontinuous hydraulic gradient across the boundary between two elements, thus provoking a non-uniquely defined flow on that interface. Some kinds of numerical methods have been developed in order to overcome the problem, as the integrated finite differences (IFD) methods and the control volume (CV)methods, both assuring good stability to the solution of transient fields of motion, and the basic conservation of mass. According to these methods, a staggered grid is introduced so to calculate a flow across lines that don’t represent a discontinuity in the hydraulic gradient. Figure 1 shows how this technique acts.

2.A TECHNICAL PATH TO PLANNING INTERVENTIONS FOR REGIONAL HYDRAULIC SAFETY DESIGN

Modelling overland flow needs a definite framework in order to get appropriate information from the analysis to be performed. In particular, a very precise insight in hydrological processes is needed, in order to estimate the flow rates in 1D streams, that originate inundation. These estimates are usually obtained through a rainfall-runoff model. Moreover, propagation of flood needs to be taken

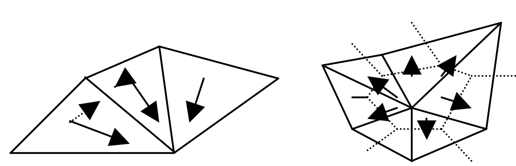

FIGURE 1.Classical finite elements layout: flow is determined by the gradient that is different for each element, and thus it is in general different in the two directions across the boundary between two elements. When control volumes are used, the grid for flow calculation and the one for the heads are staggered: no inconsistency in flows happens.

in account so to know with relative certainty the water profile in each section of the streams. This information is needed because water elevation is a critical parameter to predict the shock wave deriving from a levee failure or the rate of discharge coming from the overtopping of a levee or bank. In addition, a very detailed topographic survey is needed since in 2D flow elevations and slopes are the main driving force of propagation. It has been shown that flooding over a flat area can be regarded to as a process of reservoir filling. To describe such a process, it is of great importance to assess where in the region water is driven to go following topographic gradients, and through this to predict which are the locations where risk of flood is higher. Once the general hydrologic processes of the region, i.e. the features of the rainfall-runoff transformation and 1D flood waves propagation, are well known, and that a properly prepared (digital) elevation model is available, it is possible to use 2D propagation models to assess which are the major characteristics of the expected flood phenomena. The inflowing discharge can derive from a dam break event, but also from the overtopping of a levee or an embankment. After the modelling step is completed, a representation is available showing where and when water arrived during the flood process, which was the height it reached, and which the velocity. These basic pieces of information allow to assess where in the region risk is higher. Risk relates with the height of water for human safety and economic impact on industrial and commercial activities, but also with the velocity field in case of some particular agricultural products, for which erosion could be a strongly limiting factor. Time of propagation is of course of prime interest since it affects the possibility to put in practice safety interventions and is a major input in preparing protection and evacuation plans. The next step relates to hydraulic design and takes in account all the structural and non-structural interventions the region requires to be brought to an acceptable degree of safety. From what we pointed out, modelling of inundation through the 2D propagation equations needs to be set into a comprehensive technical framework in order to achieve good results in regional planning and safety management.

3.THE MATHEMATICAL MODEL

As well known, traditionally water waves propagation is described via the StVenant equations, that form a partial derivative system of hyperbolic type. The first one of them describes the law of mass conservation for a 2D stream, and can be written as:

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = q \qquad (1)$$

where h is the water height, t is time, x,y are the two directions of the 2D field, and u,v water velocities along them, q the source-sink term that might represent rainfall or karst phenomena.

The second StVenant equation expresses the momentum balance, and it represents the motion equation. It can be written as:

$$\frac{\partial}{\partial t}\begin{Bmatrix} hu \\ hv \end{Bmatrix} + \begin{bmatrix} hu^2 & huv \\ huv & hv^2 \end{bmatrix} * \nabla + gh \begin{Bmatrix} S_{fx} + \dfrac{\partial H}{\partial x} \\ S_{fy} + \dfrac{\partial H}{\partial y} \end{Bmatrix} = \{0\} \qquad (2)$$

in which ∇ is the gradient operator, H is the total hydraulic head and S_{fx},S_{fy} represent the friction slopes in the two directions. If we accept to neglect the first two terms in equation 2., i.e. the local and convective inertia, and furthermore we adopt the well known Manning's equation to describe the friction slopes, we finally obtain a unique parabolic-type equation as follows:

$$-div(T * \nabla H) + \frac{\partial h}{\partial t} = q \tag{3}$$

In equation (3), div is the divergence operator, T is the so-called conveyance vector, that represents the discharge per unit hydraulic gradient in the two directions, and depends on the water height, hydraulic gradient components and Manning's coefficient for roughness, n, in both directions x,y. Its components follow the expression:

$$T_x = \frac{h^{5/3}}{n_x \sqrt[4]{\left(\frac{\partial H}{\partial x}\right)^2 + \left(\frac{\partial H}{\partial y}\right)^2}} \qquad T_y = \frac{h^{5/3}}{n_y \sqrt[4]{\left(\frac{\partial H}{\partial x}\right)^2 + \left(\frac{\partial H}{\partial y}\right)^2}} \tag{4}$$

Differently from Galerkin's method, where orthogonalization conditions for each shape function were posed, the key feature of the IFD method is to impose a mass balance condition for each node, and to calculate the fluxes across the boundary of a control volume defined on the basis of geometrical features of the mesh for each node. This brings to the respect of mass balance. As a drawback, this method does not take care of the tensorial features of variables such as the conveyances (Todini et al.,1994), and thus it represents the flow field as a network of pipes. The CV method (Todini et al.,1996; Gottardi et al.,1996), on the other hand, conserves the advantages of a staggered representation of the system, as the IFD, but allows to cope with the tensorial nature of the conveyances, as one can do with the classical finite elements. This method rests on an approximation of the unknown function, the water height h(x,y), through a linear compound of "shape functions", weighted with the values of the unknown taken on the nodes of a mesh of the same type of that used in finite elements methods. The structure of the algorithm is the same as in IFD, but the use of linearly-varying functions for each element allows a smoother and less 'network-of-pipes' like representation of phenomena. A control volume can be determined for each node of the mesh using a Thiessen polygonation on a Delaunay triangulation (Todini et al.,1996); different techniques take into consideration the medianes of the triangles to construct the control volume (Gottardi et al.,1996). In the first case, geometrical considerations lead to the condition that the triangles have all acute angles so to respect the energy balance for each element, while the use of the medians should allow the use of any type of triangles. Anyway, it seems that convergence be slower in this second case, and moreover some difficulties rise when connections among 2D and 1D propagation schemes occur (Pistocchi, 1997).Equations for this method can be deduced by refering to the integral mass balance condition applied to each control volume. Once, according to the technique chosen, the control volumes have been determined, assumed to be zero the source-sink term q in equation(3), the mass balance condition *around each node* leads to a system of equations in the form:

$$\frac{\partial}{\partial t}\{h\Omega\} = \underline{\underline{T}} * \{H\} \tag{5}$$

where the vector $h\Omega$ has as many elements as the nodes in the mesh, i.e. the control volumes, and contains the product of the local control volume area, Ω, and the nodal height of water, h, thus representing the volumes stored on each node's surroundings at the generic time step, while T is the transmissivity matrix and contains the conveyance terms, and H is the vector of nodal hydraulic heads. The expression for each term in T depends on the method chosen, and usually contains a geometry-dependent parameter, being very easy to be computed in the integrated finite differences method and fairly more complex in the case of control volumes.

4. MESH CONSTRUCTION

The construction of a model of the kind described before needs to prepare a finite element type calculation mesh. As already pointed out, topographic information is very important since slopes are the main driving force of the process. Hence the need for a very good digital terrain model of which some well chosen points have to be used as nodes of the mesh itself. As a rule of thumb, the sides of the elements in a mesh should be not larger than 500:600 m if not dealing with a very flat and large region, where hydraulic gradients might be not too different from the ones resulting from such a lumping operation. Once the DTM is available, the mesh must be constructed obeying to very strict rules, i.e. the acute angles condition we already mentioned, in the case of a Thiessen polygons construction for the control volume, while it is sufficient that the triangles be fairly regular in case medians are used. Many programs exist to perform the construction of the mesh. Algorithms are under study to obtain acute triangles automatically, while standard codes only provide meshes covering a prefixed region. Hence the need for some manual adjustment of the mesh in case acute triangles are required. Other important input data are the Manning's roughness parameters for each element, that can be taken as a first guess at values of 0.04-0.08 (SI units) to take in account the high friction slope due to propagation over very irregular ground, together with the input rate of discharge. In general, it has been shown that the influence of Manning's coefficients values on the final result of the simulation is very low. They only influence the time of propagation of the overland flow.

5.CASE STUDIES

To assess the applicability of the paradigm discussed above, it has been taken in consideration the case of the Samoggia river in the province of Bologna, northern Italy. Some critical rainfall events occurred in the area during autumn 1996, showing that hydraulic safety was not granted for low probability (100 year return period) discharges. Hydrological rainfall-runoff characterisation of the area and the modelling of river flood waves propagation via the Mike 11 transient flow model were performed and critical flood waves were determined on the basis of statistical analyses (Artina et al., 1997). The information deriving from such an analysis was used to simulate an overtopping event using as input data a critical wave that showed in 1D analysis insufficient levees elevations along river Samoggia. Figure 2 shows the maps of inundation as it was simulated during a period of 45 hours . Two more cases in the area were studied, regarding the simulation of a levee break on river Samoggia occurred in autumn 1996, and the analysis of the effects of a break of the same kind at a different location. Details can be found in Pistocchi, 1997.The CV method code used to simulate these case studies, due to Todini et al., 1994, and belonging to ET&P S.R.L., Bologna, also allows a fair representation via an IFD approach of 1D flows that often occur coupled with 2D propagation, and takes in account through adequate algorithms the barriers to flow and the obstacles in 1D as well as 2D flow, that can become broad crested spillways as water surface elevation reaches levels high enough. In this case it is possible to give the phenomena a much more realistic representation than using a simple and plane finite-elements type mesh. A very important feature to be pointed out about the whole class of these models is they provide precise representations of the phenomena once the precision of a topographic survey is granted.

6.CONCLUSIONS

The need for a direct integration between the 2D model and other hydrologic analysis tools together with land use and vulnerability maps in a GIS framework should be the right point in order to achieve a good tool for impact assessment and prediction. Further efforts in such a kind of modelling should be directed to getting good topographic information and to connect a predictive tool of the type discussed to a civil protection system, so that real time forecasting might be effective in public safety planning. It must be noted that when topographic information is inadequate, other modelling tools are available (for instance the so called “accumulation cells”) that allow for a more “gray box type” representation of the region using the average height of the obstacles in a flat, homogeneous region as a parameter.

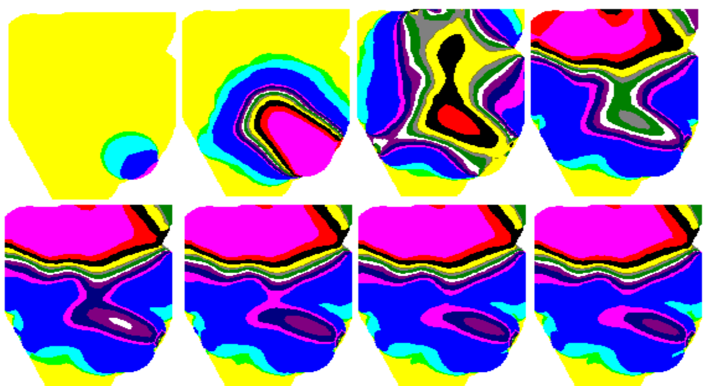

FIGURE 2:simulation of a levee overtopping

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